

# Evaluation of Diagonal Compression Strut of Masonry Infill in RC Frames Based on Experimental Investigation

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## ABSTRACT

Reinforced concrete buildings with masonry infills are widely used structural system in many countries in the world. Modelling of masonry infill using a strut model is a recommended procedure in several design codes. However, there are significant variations in estimating the strut characteristics that cause confusion for practicing engineers. Therefore, the strut width of the infill is first investigated based on the experiment results of five 1/2 scaled specimens of RC frame with masonry infill. Appropriate strut width is discussed based on comparative study with experiments and several methods in past literatures.

**Keywords :** Reinforced concrete, Masonry infill, Quasi-cyclic loading, Compression strut width.

## 1. INTRODUCTION

Many of the buildings, which experienced damage in the recent earthquakes such as 2015 Nepal Earthquake, 2016 Ecuador Earthquake and 2017 Mexico Earthquake, were reinforced concrete buildings having partitions of masonry walls. Those masonry partitions walls were commonly considered as non-structural elements and the structures were designed as RC moment resisting frames ignoring their influences. However, masonry infill walls can completely change the behavior of structures as noted by many researchers in several experiment studies such as Paulay and Priestley [1]. In general, masonry infill increases the frame strength, which can be considered as a beneficial point. On the other hand, masonry infill can completely change the failure mode of RC frames due to the additional moment and shear forces exerted by the infill. In addition, masonry infill greatly increases the structure stiffness that significantly changes the natural period of a building and thus changes seismic demand. The irregular positioning or/and eliminating several masonry infill walls in a building floor can cause torsional forces or soft story collapse as was noticed in past earthquake experiences as mentioned in several references [1~3].

Even though masonry infills were experimentally and analytically studied by several researchers, there are still large variations in recent design codes on the methods to evaluate seismic capacity of masonry infill [1~4]. The most recognized method to model the masonry infill is using a compression equivalent strut that is used as a truss element added to the RC frame model. This method is popular among practicing engineers because of its simplicity and less computational time and effort. However, literature review shows large variations between the proposed methods to estimate the strut width,  $W_{inf}$ . This variations

cause confusion for practicing engineers on which strut width is appropriate for modelling. Therefore, the main objective of this study is to evaluate the appropriate range of strut width based on experimental studies of five 1/2-scale single story reinforced concrete frames with masonry infill and comparative studies with several proposed methods in past literatures.

## 2. TESTS PROGRAMS

### 2.1 Test specimens and parameters

Several parameters might greatly influence the seismic performance of masonry infill such as the masonry type, panel aspect ratio, mortar characteristics and strength, frame strength, vertical load and openings. In this study, five half-scaled specimens were designed based on three main parameters: i) varying the ratio of lateral strength of RC column to masonry infill, ii) varying the strength and stiffness of RC beam (very rigid strong beam and relatively flexible beam), iii) varying the strength of mortar of infill panel, the summary of parameters is shown in Table 1.

Those three parameters were chosen based on the investigation of several variances in existing RC buildings in Bangladesh [5]. The case study of Bangladesh is considered, since this experimental study is a part of a wider scope ongoing experimental program of a Japanese project called SATREPS [6], which intended to upgrade seismic evaluation methods of reinforced concrete buildings in Bangladesh.

To classify the frame into weak and strong ones, the  $\beta$  index is used, which is defined in this study, as shown in Eq. 1.

$$\beta = V_f / V_{inf} \quad (1)$$

Where  $V_f$  is the boundary frame lateral strength which is calculated to be the ultimate flexural capacity of a bare

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Table.1 Summary of specimen details and varying parameters

Series no.	Main varying parameter	Specimen name	$\beta$ index	Column dimension (mm)	Beam details (mm)	Ratio $M_{ub}/M_{uc}$ *	Mortar strength
Previous study	Strength of RC columns	F-1.5	1.51	300 x 300 (Ref [8])	(bxd) 600 x 400 Main bars; 10D-22 Stirrup; D13@100	3.3	Strong
		F-0.4	0.39	200 x 200 (Ref. [8])		9.7	Strong
		F-0.6	0.56			5.9	Strong
This study	Weak beam	WB	0.43	200 x 200 Main bars; 4-D16 Hoop; D10 @50	(bxd) 200 x 250 Main bars; 6-D13 Stirrup; D6 @100	0.7	Strong
	Weak mortar	WM	0.82		600 x 400	5.9	Weak

\* $M_{ub}/M_{uc}$  is the ratio of beam to column's plastic moment capacity

frame with plastic hinges at top and bottom of columns.  $V_{inf}$  is the masonry infill lateral strength calculated based on Eq. 2 that is a simplified empirical equation showing good agreement with previous experimental database studied by the author [7].

$$V_{inf} = 0.05 f_m \cdot t_{inf} \cdot l_{inf} \quad (2)$$

Where  $f_m$  is the compressive strength of masonry prism,  $t_{inf}$  is the infill thickness and  $l_{inf}$  is the infill length.

Specimens named F-0.4, F-0.6 and F-1.5 represent the ratio of lateral strength of frame to masonry infill,  $\beta$  index, of 0.4, 0.6 and 1.5, respectively. The only difference is the strength and reinforcement of RC columns. The beams were designed to be strong and stiff enough to simulate a typical case of a weak column and strong beam system, observed in existing buildings in Bangladesh designed by old standards. Details of specimen F-0.6 is shown in Fig.1. The details and experiment results of two specimens (F-1.5 & F-0.4) were presented in the previous study by the author [8].

The specimen named WB (weak beam) was designed with weak beam and strong column, in order to study the influence of this parameter on the seismic capacity and collapse mechanism of infilled RC frames. Specimen WB is designed to be exactly identical to specimen F-0.6 except for the upper beam. The upper beam is relatively smaller and designed to have a ratio of beam plastic moment capacity ( $M_{ub}$ ) to the column plastic moment capacity ( $M_{uc}$ ) of 0.7 ( $M_{ub}/M_{uc}=0.7$ ). The details of specimen WB are shown in Fig.2.

Masonry infill walls in many countries are considered as non-structural walls and thus low quality mortar strength might be commonly used. Therefore, the specimen named WM (weak mortar) was constructed using masonry walls with very low compressive strength mortar in order to study its influence on seismic capacity. Other than the mortar strength, the specimen WM was designed exactly same as the specimen F-0.6.

## 2.2 Test setup and loading protocol

The loading system is shown schematically in Fig.3. The vertical load was applied on RC columns by two vertical hydraulic jacks and was maintained to be 200kN on each column. The common construction practice is that masonry infill is inserted (infilled) after the construction of RC frames. Several past studies[1], recommended that most of the gravity load already taken by columns. Therefore, vertical load is applied directly to the columns. Two pantograph, attached with the vertical

jacks, restricted any torsional and out-of-plane displacement. Two horizontal jacks, applying together for an incremental cyclic loading, were attached at the beam level and were controlled by a drift angle of  $R\%$ , defined as the ratio of lateral story deformation to the story height measured at the middle depth of the beam ( $h=1,600\text{mm}$ ). The lateral loading program consisted of 2 cycles for each peak drift angle of 0.05%, 0.1%, 0.2%, 0.4%, 0.6%, 0.8%, 1%, 1.5% and 2%. Specimens, which didn't significantly degrade in strength after the final cycle of 2 %, were then pushed monotonically until severe damage were observed.

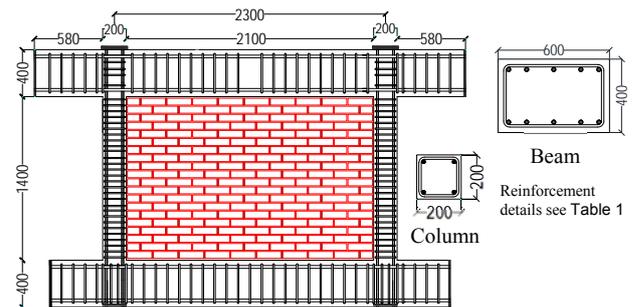


Fig.1 Details of specimen F-0.6&WM; units in mm

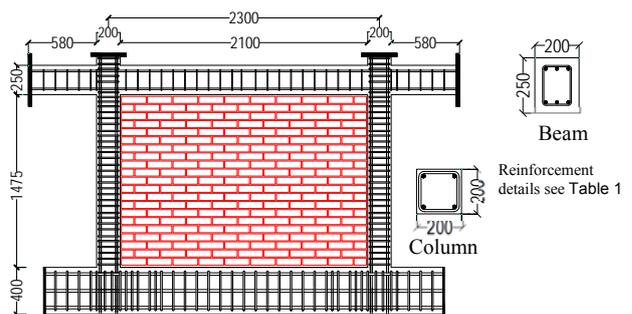


Fig.2 Details of specimen WB; units in mm

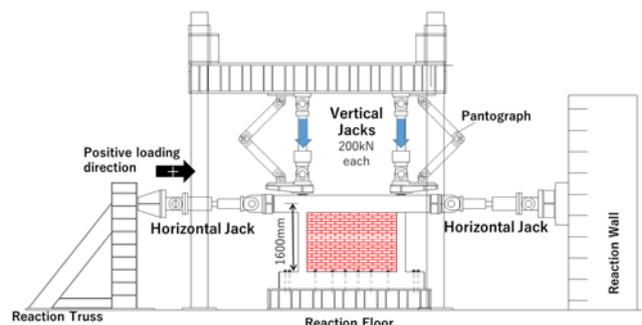


Fig.3 Schematic figure of test setup

### 2.3 Material properties

The infill panels are constructed using 60x100x210 mm (height x thickness x length) solid bricks. A professional mason built the infill after the frame construction, where its thickness is 100mm and mortar head and bed joint thickness is about 10mm.

Table 2, Table 3 and Table 4 show the material mechanical properties of concrete, masonry and reinforcing steel, respectively. The material tests of steel and concrete were performed according to the Japanese material standard [9]. The masonry prism compressive strength was tested as per the ASTM C1314 [10]. The concrete used for all specimens had the same mix design. The proportion of cement and sand for the mortar is 1:2.5 (mass proportion) for all specimens, except for specimen WM with weak mortar where 1:6 is used. The material tests were conducted at the same time with the experimental loading for each specimen individually. There was a slight difference in compressive strength of concrete, mortar and masonry prism which is thought to be due to the time gap between loading tests.

Table.2 Concrete properties

Name	Compressive strength (MPa)	Elastic modulus (MPa)	Split Tensile strength (MPa)
F-1.5	28.3	$2.71 \times 10^4$	2.45
F-0.6	25.5	$2.42 \times 10^4$	1.94
F-0.4	24.2	$2.31 \times 10^4$	2.06
WB	23.6	$2.37 \times 10^4$	1.96
WM	25.8	$2.42 \times 10^4$	2.03

Table.3 Masonry properties

Name	Prism Compressive strength (MPa)	Prism Elastic modulus (MPa)	Mortar Compressive strength (MPa)	Brick unit Compressive strength (MPa)
F-1.5	18.6	8140	29.2	38.1
F-0.6	19.5	10230	27.7	
F-0.4	17.3	7840	20.2	
WB	19.5	10230	27.7	
WM	13.3	5470	4.8	

Table.4 Reinforcement mechanical properties

Bar	Nominal strength	Yield strength (MPa)	Ultimate tensile strength (MPa)
D6	SD345	346	546
D10	SD345	384	576
D13	SD345	380	568
D16	SD345	380	563
D22	SD390	447	619

### 2.4 Experimental Results

As explained earlier, this study presents a brief outline of the experimental results which are related to the main objective of evaluating strut width of infill. The lateral load versus story drift angle of specimens F-0.6, WM and WB are shown in Fig. 4. The results of

specimen F-1.5 and F-0.6 were presented in a previous study of the author [8].

For all specimens (except for specimen WM), very small cracks on mortar bed joint and diagonal cracks on bricks near the loading corner, less than 0.1mm width, started to develop at early stages of loading just when the drift angle was 0.05%. The cracking of both infill and RC frame was clearly visible at 0.1% for all specimens.

As for specimens F-0.6, WM and WB, the longitudinal reinforcement in columns yielded at drift angles between 0.6%~0.8%, which are also approximately the point when the maximum strength is recorded (except for the negative loading of specimen WM which showed relatively greater ductility behavior).

Based on these experiment results, the compression strut width of the infill is discussed in detail in next chapter.

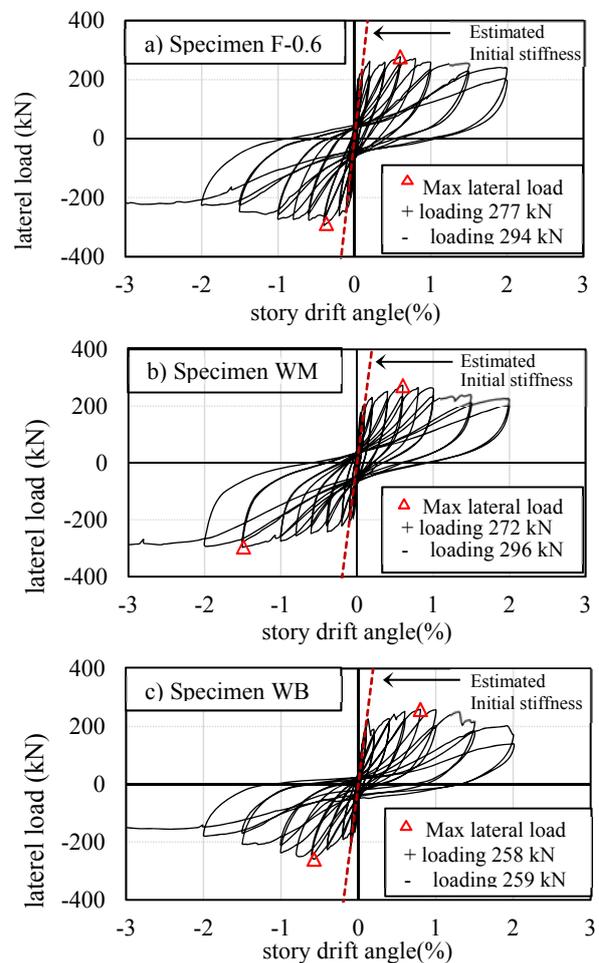


Fig.4 Lateral strength and story drift angle relations for specimens: a) F-0.6 b) WM c) WB

## 3. STRUT WIDTH AT INITIAL STIFFNESS

### 3.1 Initial stiffness based on experimental results

The initial stiffness of infilled frame is taken as the slope between the origin point of the load-displacement curve and the point in which there is a cracking starts to be easily visible in the masonry infill and the RC frame. Significant degradation of stiffness is

also considered to decide the point, which was determined to occur at story drift of 0.1% as shown dotted line of estimated initial stiffness in Fig.4. Even though the investigated parameters influenced strength and deformation capacity of specimens, but those parameters didn't significantly influence the initial stiffness as shown in backbone curves for story drift 0~0.1% in Fig.5.

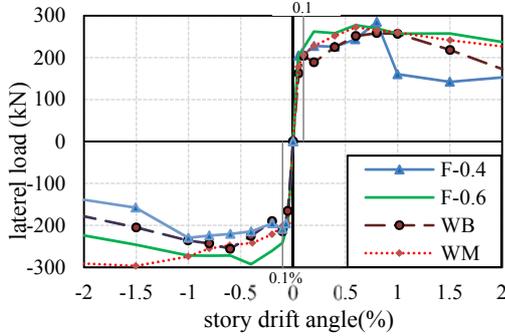


Fig.5 Backbone curves and initial stiffness of specimens: F-0.4, F-0.6 WB,&WM

### 3.2 Overview of existing methods in literature

RC frame with masonry infill is an ongoing research since 1960's, but there is still a significant variation in existing proposed methods in estimating the initial stiffness. The most common method is using the concept of diagonal compression strut, but the appropriate strut width is a controversial topic among many researchers. In this study, recent and some well-known design codes or methods that are commonly cited on this topic are briefly introduced and investigated.

#### (1) ASCE/SEE 41(2006)

ASCE/SEE 41 [2] adopted a method to calculate the diagonal compression strut width at initial stiffness based on empirical relation on the ratio stiffness of frame to masonry infill, as shown in Eq. 3 and Eq. 4.

$$W_{inf} = 0.175 \cdot (\lambda_h H)^{-0.4} \cdot d_m \quad (3)$$

$$\lambda_h = \left[ \frac{E_{inf} t_{inf} \sin(2\theta)}{4E_c I_c h_{mf}} \right]^{\frac{1}{4}} \quad (4)$$

Where  $W_{inf}$  is the equivalent strut width,  $t_{inf}$  is the infill thickness (see Fig 6),  $E_{inf}$  and  $E_c$  are the elasticity moduli of the infill and the concrete.  $h_{inf}$  and  $H$  are the net height of the infill and the story height.  $\theta$  is the diagonal inclination angle of the infill.  $I_c$  is the moment of inertia of the RC column,  $d_m$  is diagonal length of infill.

#### (2) New Zealand seismic assessment standard of existing buildings (2017).

The NZ standard [3] used a modified version of Eq. 3, as shown in Eq. 5.

$$W_{inf} = 0.18 \cdot (\lambda_h \cdot H)^{-0.25} \cdot d_m \quad (5)$$

#### (3) Masonry society joint committee (2016)

The MSJC [4] proposed a similar concept with ASCE/SEE 41 [2] where the strut width is related to the empirical ratio of stiffness  $\lambda_h$  in the previous Eq. 4, but proposed another method for strut width as in Eq. 6.

$$W_{inf} = 0.3 / \lambda_h \cdot \cos \theta \quad (6)$$

#### (4) Paulay and Priestley (1992)

Paulay et al. [1] used a simplified assumption as shown in Eq. 7 where the strut width at initial stiffness is

not related to the ratio of frame to infill stiffness or strength.

$$W_{inf} = 0.25 \cdot d_m \quad (7)$$

#### (5) Fiorato et al.(1970)

Fiorato et al. [11] proposed a method assuming the structure to be a composite beam with RC columns, where the beam and columns are the flanges and the masonry wall is the web.

$$K_{ini} = \frac{1}{(1/K_{sh}) + (1/K_{fl})} \quad (8)$$

$$K_{sh} = A_{inf} \cdot G_{inf} / h_{inf} \quad (9)$$

$$K_{fl} = 3E_c \cdot I / h^3 \quad (10)$$

In Eq. 8, Eq. 9 and Eq. 10, it is noted that for the calculation of shear stiffness,  $K_{sh}$ , only the masonry wall panel is considered where  $A_{inf}$  is the cross-sectional area of infill and  $G_{inf}$  is the shear modulus taken as  $0.4E_{inf}$ . For the calculation of flexural stiffness,  $K_{fl}$ , the whole composite section is used. Here,  $I$  is the equivalent moment of inertia of the transformed section considering the elastic moduli of concrete and masonry infill. This method does not use the concept of compression strut, but this method is included in this study, since many recent studies used it to estimate initial stiffness.

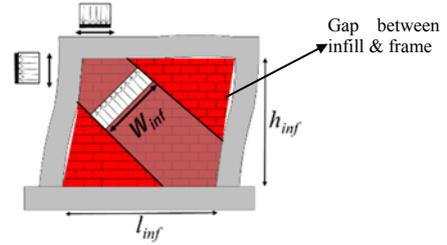


Fig.6 Effective strut width at maximum strength

### 3.3 Comparative study

The strut width based on the experimental studies and existing models are compared in Fig.7, where the strut width  $W_{inf}$  is normalized by diagonal length  $d_m$  to simplify the comparison.

It is noted that the initial stiffness of the experiment and the method of Fiorato [11] is transformed into equivalent strut width based on Eq. 11 and Eq. 12. Herein,  $K_{ini}$  is the initial stiffness obtained from the summation of the initial stiffness by RC frame ( $K_{RC,frame}$ ) and the masonry infill ( $K_{strut}$ ).  $K_{strut}$  is calculated using the axial stiffness of equivalent diagonal strut as shown in Eq. 12. The initial stiffness of RC frame,  $K_{RC,frame}$ , is calculated by theoretical equations by the reference [12] for elastic stiffness of bare frame, as shown in Eq. 13 and Eq. 14.  $I_c$  and  $I_b$  are the moment of inertia of RC column and beam, respectively.

$$K_{ini} = K_{strut} + K_{RC,frame} \quad (11)$$

$$K_{strut} = \frac{E_{inf} \cdot W_{inf} \cdot t_{inf} \cdot \cos^2 \theta}{d_m} \quad (12)$$

$$K_{RC,frame} = \left( \frac{24E_c I_c \cdot 12\rho + 1}{h_c^3 \cdot 12\rho + 4} \right) \quad (13)$$

$$\rho = \frac{\sum E_c I_b / l_b}{\sum E_c I_c / h_c} \quad (14)$$

The strut width of masonry infill, based on the experimental study at initial stiffness, ranged between  $0.15d_m \sim 0.3d_m$  with an average of  $0.2d_m$ . The ASCE/SEE41 [2] and MSJC [4] greatly underestimate the strut width by more than the half. On the other hand, Fiorato's method [11], greatly overestimates the initial strut width. Based on the experimental results in this study, the best existing methods to estimate the strut width were those of Paulay et al. [1] and NZ [3], which represent the upper and lower boundary of the strut width, respectively. In addition, there was no clear relation between frame to infill stiffness ratio and the strut width. However, the infill made by weak mortar showed an increase in the strut width.

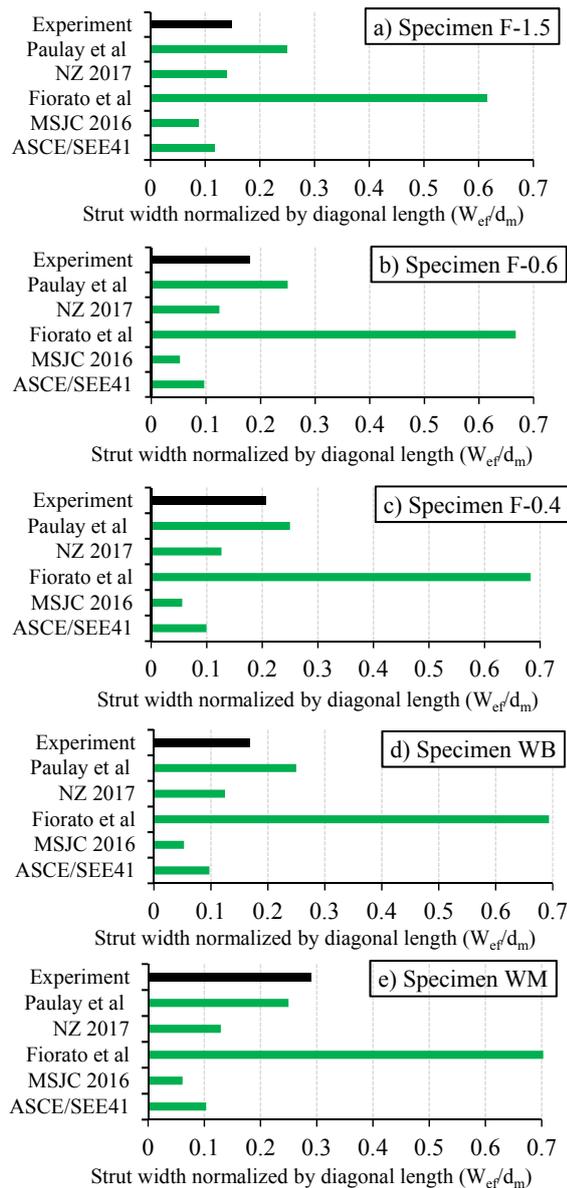


Fig.7 Comparison of strut width based on experiments and existing methods

#### 4. STRUT WIDTH AT MAXIMUM STRENGTH

Next, the strut width at the maximum strength is discussed. To calculate the strut width of masonry infill at the maximum strength, first, the strength of the infill and frame should be separated. In this study, the

maximum lateral load of the masonry infill in the experiments is calculated by subtracting the bare frame lateral strength on the maximum experimental lateral load, as shown in Eq.15. Calculating the actual maximum lateral strength of the infill can be more complicated, due to the complex frame-panel interaction. In the experiments, the maximum load occurred at about drift story of 0.8%. At this drift point, the RC columns have already yielded at about 0.6%, which was measured from strain gauge values. Therefore, subtracting the bare frame strength is thought to be acceptable assumption, Eq.15 and Eq.16 is employed as a comparison benchmark and the results are shown in Table 5.

$$V_{inf} = V_{max} - V_f \quad (15)$$

$$V_f = 4M_u/H_{cl} \quad (16)$$

Where  $V_{max}$  is the maximum lateral load of overall frame,  $M_u$  is the minimum plastic moment of either the column or beam calculated by AIJ provision [13].  $H_{cl}$  is the clear height (taken here as infill height).

Table.5 Experimental lateral strength of infill

Specimen	Experiment $V_{max}$ (kN)		$V_f$ (kN) Eq.17	Avg. $V_{inf}$ both directions (kN)
	Positive loading	Negative loading		
F-0.4	285	230	71	186.5
F-0.6	277	295	113	173
F-1.5	571	582	280	296.5
WB	258	259	88	170.5
WM	272	296	113	171

The strut width of masonry infill at the maximum strength is not the same as that at initial stiffness. The main reason is that masonry infill and RC frame get detached and a gap is formed with the increase of the lateral load, which is due to the deformation of the frame and panel (see Fig.6). In other words, the weaker is the frame (more flexible), the greater is the deformation and the gap increases. In addition, the compressive stress distribution in the strut is not uniform having a shape of parabolic or triangular distribution with the maximum stress at the corner. For simplicity and modeling purposes, the effective strut width is taken as uniform stress, as shown in Fig.6, and it was obtained as below.

The strut width at maximum lateral strength of masonry infill,  $V_{inf}$ , can be calculated using Eq. 17. This is a common method used by several references such as in FEMA 306 [14].

$$V_{inf} = W_{inf} \cdot t_{inf} \cdot f_{m90} \cdot \cos \theta \quad (17)$$

Where  $f_{m90}$  is the expected prism compressive strength of masonry in horizontal direction, which may be set as 50% of the expected prism compressive strength.  $f_{m90}$  is used instead of normal prism compressive strength,  $f_m$ , because masonry infill is a non-isotropic material and the compressive strength depends on load direction, as shown in the experiments results of Page [15].

The strut width at maximum strength and at initial stiffness for the five specimens are then compared in Fig.8. The strut width at maximum strength is about 40~60% of those at initial stiffness except for specimen

F- 1.5 which is the case of very strong frame where strut width didn't significantly change. This is considered to occur because stronger frames have less flexural deformation and thus the contact length between infill and frame was not significantly changed.

Fig.9 shows the relation between  $\beta$  index (the relative expected strength of frame to masonry infill as shown in Eq. 1) and the strut width at maximum strength. Those plots are based on the experimental results of this study and 16 specimens by other researchers, which are summarized in Table 6. Further specimen details by other researchers are presented in the reference [8]. As shown in Fig.9, as the frame strength increases the strut width tends to increase, and there is a proportional tendency between these two factors. Based on this relation, lower boundary for the strut width at maximum lateral strength is proposed, as shown in the Fig.9, which will provide reasonable conservative values to predict the maximum strength of the overall frame.

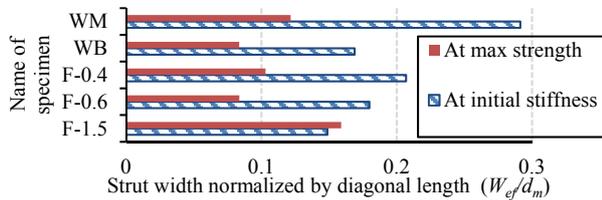


Fig.8 Comparison of strut width at initial & max strength

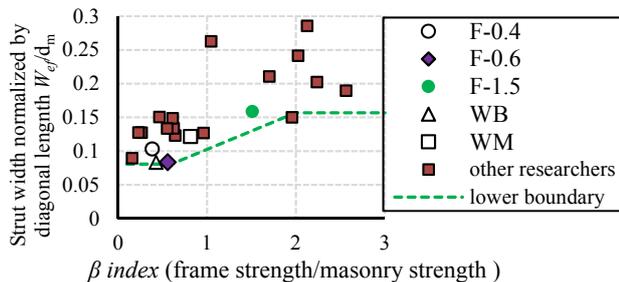


Fig.9 Relation of strut width and  $\beta$  index

## 5. CONCLUSION

The followings are the main conclusions:

- Strut width of masonry infill at initial stiffness ranged between 0.15 and 0.3 times its diagonal length. There are large variations between different codes, and the methods of Paulay and Priestley [1] and NZ [3] could represent the upper and lower boundary of the strut width, respectively, for the specimens employed in this study.
- There was a reduction of about 40%~60% of strut width at maximum strength when compared to those at initial stiffness, in the case of relatively weak frames. This is considered to be mainly due to the increase of separation between infill panel and RC frame at higher loads.
- A relation was found between the frame strength and the strut width of masonry infill. Based on this relation, lower boundary for strut width at maximum strength is proposed. The proposed lower boundary is useful for practicing engineers to easily evaluate the strength and to model the masonry infilled-RC frame with some conservativeness.

Table.6 Investigated past experimental data

Researcher	Specimen	Type of Infill	$\beta$
Mehrabi et al	3	solid bricks	0.64
	5	solid bricks	0.62
	7	solid bricks	1.05
	9	solid bricks	0.62
	11	solid bricks	0.55
Jin et al	IFRB	concrete block	2.24
	IFFB	concrete block	1.96
T. Suzuki et al	1S-1B	concrete block	0.96
D. Kakaletsis et al	S	bricks (hollow)	3.93
B. Blackard et al	S	brick(two-wythe)	0.16
H. AlNimry	IF4	stone & concrete	0.27
	IF5	stone & concrete	0.23
Imran et al	Model 1	AAC blocks	2.13
	Model 2	brick	1.70
Zovkic et al	Model 8	brick	4.04

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