- Technical Paper-

OPTIMIZATION OF NUMBER AND LOCATION OF ACCELEROMETERS FOR MONITORING STRUCTURES

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ABSTRACT

The location of sensors to all floors is preferable to estimate displacement in structural health monitoring system; but not a practical solution because of limited accessibility, e.g. Fukushima nuclear power station (NPS). In this paper, the optimization of the number and location of sensors applied to NPSs is evaluated with the effect of stiffness degradation on the response. Linear and mode estimation methods are used. The mode method using 2 sensors show error in the displacement estimation lower than 15%, while for both methods the error increases to 30% if the stiffness degradation is considered.

Keywords： Nuclear power stations, optimization of sensors, structural response, mode interpolation

1. INTRODUCTION

Structural health monitoring (SHM) of nuclear power stations (NPS) in Japan has become an important technology recently. One type of SHM system aims to quantify drift demand estimated using measurements obtained from in-situ accelerometer sensors because drift is one of the most critical parameters to identify structural damage after an earthquake [1,2]. In some cases, placing a sensor on each floor may be difficult and impractical. When considering NPSs, an optimal location of sensors is necessary to increase the accuracy of quantifying the level of damage of vital infrastructure.

To prevent structural failures in future earthquakes, a monitoring strategy to estimate damage must be implemented for NPSs. For example, after the 2011 Great East Japan earthquake, the Fukushima NPS suffered severe damage. Therefore, the motivation of this study is to 1) optimize the number and location of accelerometer sensors applied to a structure model similar to the Fukushima NPS and 2) investigate the effect of structural damage on the optimal location of sensors with the assumption that some floors have experienced stiffness degradation.

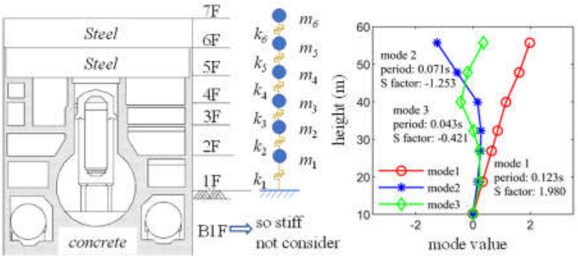
2. BACKGROUND AND METHODOLOGY

Previous research has focused already on estimating the maximum displacement of structures using a limited number of sensors. Xu and Akira [1] proposed a mode-based method for estimating floor displacement, as well as Kikuchi et al. [3] and Pan et al. [4]. In these methods, the mode was assumed to remain constant with changes in the fundamental frequency, and no change in mode shape with stiffness degradation or structural damage was considered. Also, there is no current detailed analysis on optimizing the location and number of sensors applied to the NPSs.

This paper evaluates the effect of the location and number of sensors for a proposed NPS model. Linear and mode methods are used to estimate displacements. Several recommended number and location for this case are proposed. After that, the post-optimized number and location of sensors are applied to some cases assuming damage on a selected floor by considering stiffness degradation of 80% and 60%. The error is compared with an original model to check the effect of stiffness degradation on the estimated accuracy of linear and mode methods.

2.1. NPS building model

A view of an NPS is described in Fig. 1(a). The structural parameters of the building obtained from open documents [5] are listed in Table 1.



(a) (b)

Fig.1 NPS model (a) front view (b) modes

Table 1. Key parameters of an NPS model [5]

Floor	Mass (t)	Height (m)	Shear stiffness (10^7 KN/m)	Effective area (m^2)
7F	1900	7.9	2.86	21
6F	1600	7.9	3.82	28
5F	7500	7.6	14.49	103
4F	8800	5.4	29.88	151
3F	11000	8.2	26.63	204
2F	13000	8.5	28.52	227

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The first three modal responses of the structure are shown in Fig. 1(b). In this model, the base floor (BIF) is assumed to have a large stiffness compared with other floors, and thus it is assumed as a fixed structure at 1F. No rocking or sway deformation of the base is considered in this study.

2.2. Methods used for optimization

Double integration after band-pass FFT filtration (2Hz to 20Hz) of acceleration data is carried out to obtain the displacement of each floor. The most common method to obtain the displacement of floors without an accelerometer sensor is to assume a displacement distribution over the story height, so the unknown displacement can be estimated by interpolation.

A simple approach is a linear interpolation where the displacement distribution is a straight line, as shown in Fig 2. The displacements of floors without data are generated by a linear interpolation.

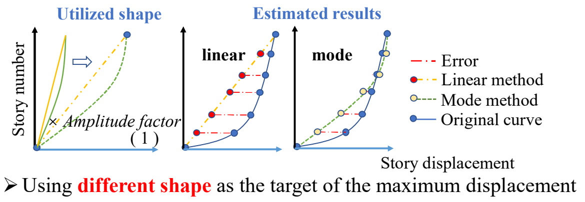


Fig.2 Linear and modal estimation method

This linear interpolation assumption is simple and accurate when the mass and stiffness distribution is nearly uniform. However, such an assumption will lead to larger errors when there is mass and stiffness discontinuities over the height of the structure.

Another method computes the displacement of each floor (d_i) by calculating the mode shape (Fig. 2) using modal coordinates estimated from Eq. 1 [6]:

$$d_i = \sum_{j=1}^{n} \varphi_{ij} H_j \qquad (1)$$

where i represents the floor number, j represents the modal order, φ_{ij} is the mode shape value of order j in floor i, and H_j is the modal participation factor of order j. If the structure is stiff enough, such as NPSs, the participation of higher-order modes is small, and ignoring all but the first mode leads to a satisfactory result [1, 4]. Hence, only the first mode is considered in this paper. According to this assumption, Eq. 1 can be rewritten as Eq. 2.

$$d_i = \varphi_{i1} H_1 \qquad (2)$$

From Eq. 2, the displacement distribution is the product of the first mode shape and the fundamental mode participation factor (H_1). Because H_1 is constant, it can be calculated as shown in Eq. 3:

$$H_1 = {}^{d_e}\!/_{\varphi_{e1}} \qquad (3)$$

where e represents the floor number of each known floor. The fundamental mode participation factor may be obtained from the data associated with each known floor assuming the fundamental mode of the structure is known. H_1 can be used in Eq. 2 to estimate the displacement of each floor. The process of estimating displacements using both linear and mode methods is illustrated in Fig. 2.

If there are 2 sensors only, the factor H_1 can be estimated uniquely as suggested by Fig. 3. But if there are more than 2 sensors, there will be multiple values of H_1 because of differences in accuracies of sensors and the effect of higher modes on structural response as shown in Fig. 4. In the latter case, each known displacement is used to compute H_1 in Eq. 3. The minimum estimated value (H_{1min}) and the maximum estimated value (H_{1max}) are determined. The value of H_1 is set to shift smoothly from H_{1min} to H_{1max}. For each value of H_1, the estimated error associated with H_1 at each floor is calculated using Eq. 4.

$$er_e = \left| {}^{(\varphi_{e1} H_1 - d_e)}\!/_{d_e} \right| \qquad (4)$$

where d_e is the displacement of floor e. er_e is the estimated error of floor e. A combined error of all locations is computed using Eq. 5. Assuming the displacements of floor e, f, and g are known, the combined error would be:

$$erc = \sqrt{er_e^2 + er_f^2 + er_g^2} \qquad (5)$$

The estimation of error is shown in Fig. 3. The most accurate value of H_1 shall have the minimum combined error as shown in Fig. 4.

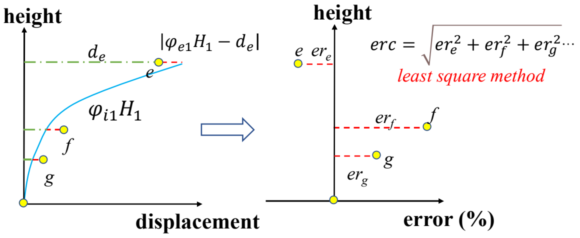


Fig.3 Determination of combined error

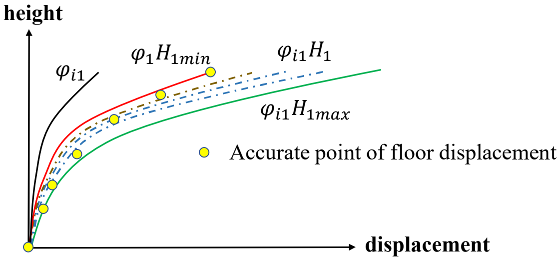
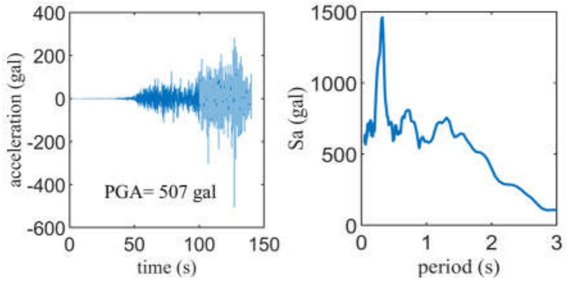


Fig.4 Determination of H_1 in mode estimation

3. ANALYSIS FOR NUMBER AND LOCATION

The EW component of the ground motion recorded during the 2011 Great East Japan Earthquake [5] is used to analyze the mentioned NPS structure. A parametric study of the number and location of sensors is carried out based on the acceleration response of each floor. Because the NPS shown in Fig. 1 was designed to remain elastic, story drift ratios smaller than 0.2% were observed for each story when using the input record shown in Fig. 5. The first mode value was obtained directly from from a modal analysis using the building characteristics listed in Table 1. A study is carried out based on the original model in Table 1. The damping ratio is selected as 5%. Damage is assigned to some floors to investigate the influence of damage on NPSs, such as the case of the Fukushima NPS after the 2011 Great East Japan Earthquake.

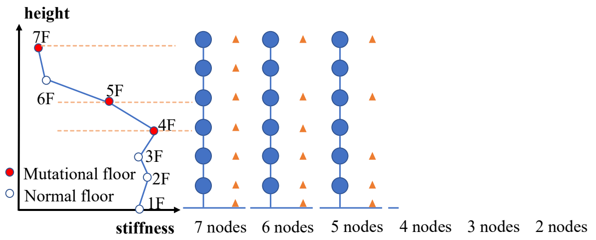


(a) (b)

Fig.5 Input record (a) time history (b) acceleration spectrum

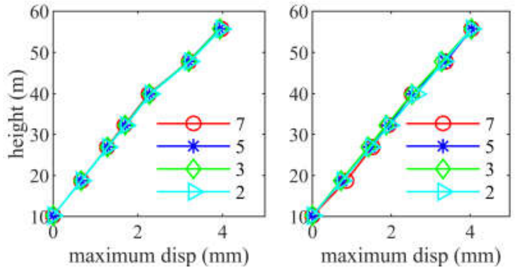
3.1. Effect of number of sensors

The location of sensors in the structure with various levels of damage and varying numbers of sensors is selected based on the stiffness distribution shown in Fig. 6(a), where the distribution can be divided into 3 regions: stories 1-3, story 4, and stories 5-6. The possible locations of sensors is considered near the stiffness discontinuity. Based on this rule, the locations under a different number of sensors are assumed for several cases, as shown in Fig. 6(b).



(a) (b)

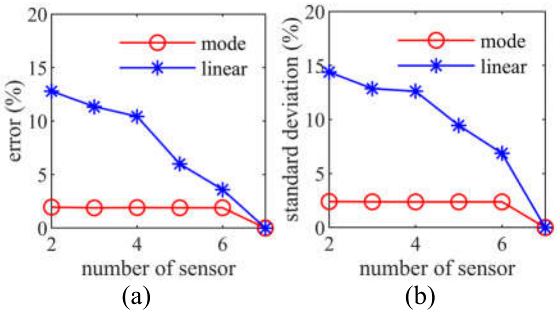
Fig.6 Conditions for number optimization (a) choose method (b) assumed cases (groups)



(a) (b)

Fig.7 Estimated maximum displacement using different methods (a) mode (b) linear

Two methods (linear interpolation method and mode shape interpolation method) explained in Section 2.2 are used to estimate the displacement response of floors without sensors. The results associated with different numbers of sensors are shown in Fig. 7. Average error computed using Eq. 6 is carried out. For each condition, the standard deviation (SD) of the error of each floor is calculated to show the discrete degree.



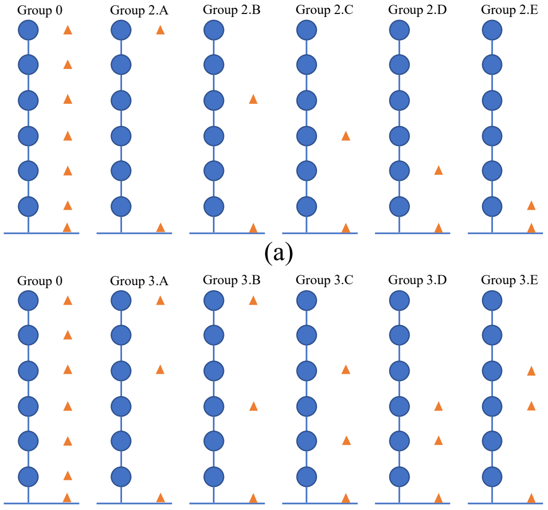
(a) (b)

Fig.8 Error associated with different numbers of sensors (a) error (b) standard deviation (SD)

$$er'_n = \left|Es_n - Es_{n-1}) - (d_n - d_{n-1})\right| \Big/ (d_n - d_{n-1}) \quad (6)$$

where er'_n, Es_n and d_n are the error, estimated displacement, and displacement measured by sensor in floor *n*.

The average error of floors without sensors computed using Eq. 6 is shown in Fig. 8. The error and standard deviation decreases with increases in the number of sensors for the linear method indicating that the accuracy of linear estimation increases with the number of sensors. But the error was nearly constant (approximately 2%) for the mode method suggesting that having fewer sensors does not lead to an increase in error using said method. It should be noted that the mode method for the given NPS model results in small errors because the structure operates in the elastic range of response.



(a)

(b)

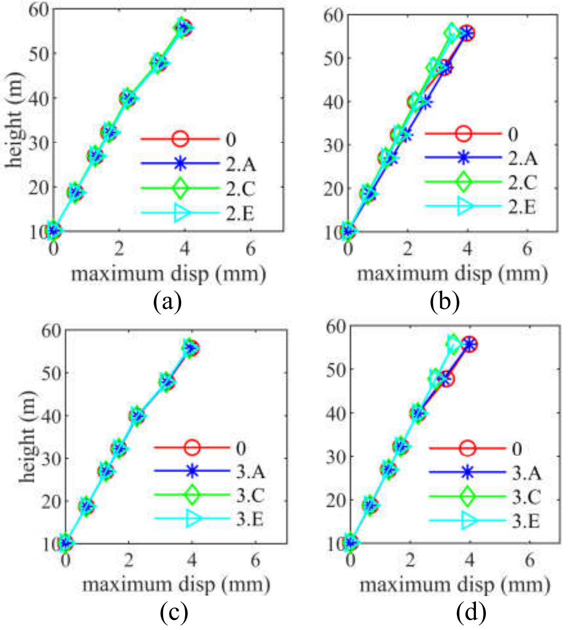
Fig.9 Setting of different condition for location optimization (a) 2 nodes (b) 3 nodes

Fig. 8 shows that the error associated with two sensors is smaller than 15% using the linear method, which is also small for practical purposes [1]. Hence, based on the result of this section, two and three sensors are selected as the optimal numbers. The investigation of the optimal location of these two and three sensors is discussed in the following section.

3.2. Effect of location of sensor

The analysis of the optimum location of sensors is carried out in this section. For the two and three sensors, several locations are considered, as shown in Fig. 9(a) and 9(b). The locations are selected based on the stiffness distribution explained in Section 3.1.

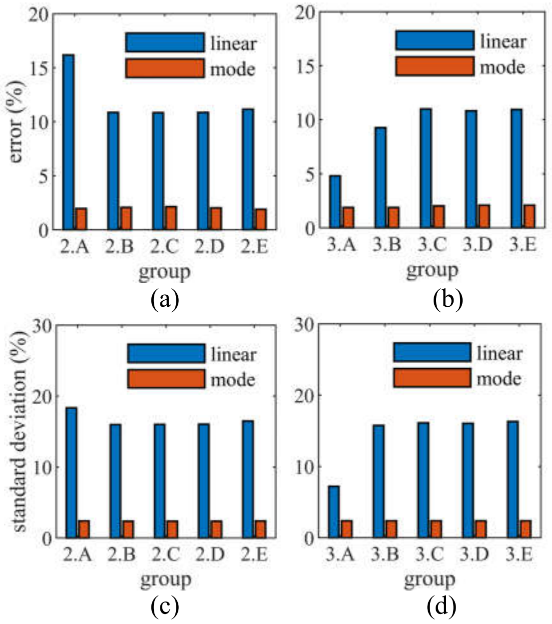
The estimated results using linear and mode methods with different locations of sensors are shown in Fig. 10. Error analysis is then applied to the results of these two methods, and the results are shown in Fig. 11.



(a) (b)

(c) (d)

Fig.10 Estimated maximum displacement with sensors in different locations (a) mode-2 nodes (b) linear-2 nodes (c) mode-3 nodes (d) linear-3 nodes



(a) (b)

(c) (d)

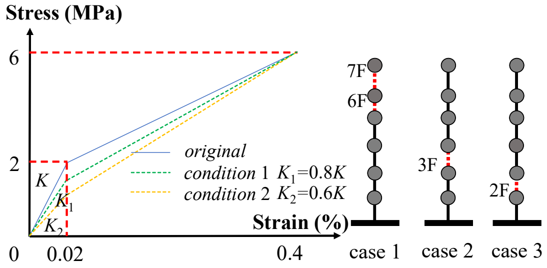
Fig.11 Error analysis (a) 2 nodes error (b) 3 nodes error (c) 2 nodes SD (d) 3 nodes SD

Fig. 11 shows that the error and standard deviation were nearly constant at 2% for the mode method for each group, which shows great result for the first-mode estimation when the mode is exactly accurate. For the linear estimation, group 2.A has an average error larger than 15% because of the large error in the middle floor. In the case when the sensor is attached to the middle floor (group 3.A and 3.B), the error decreases about 2 times, as shown in Figure 11(b). The standard deviation also shows a similar trend for linear estimation. Hence, groups 2.B, 3.A and 3.B are selected as the best location for further investigation in the next section. As noted, the mode shape method using the NPS model gives a low error, since the mode shape does not change as the building remains in the elastic range.

4. ANALYSIS CONSIDERING THE EFFECT OF STIFFNESS DEGRADATION

In this section, the accuracy and applicability of the described methods are studied in the case where there are differences between the stiffness distribution of the analytical model and real building because of prior damage and deterioration. Based on the optimal location and number of sensors obtained in previous sections, several cases of stiffness degradation are assumed, as shown in Fig. 12(a).

Stiffness degradation is considered here but changes in ultimate strength as a result of prior damage are not. This assumption is based on an experimental study [2,7] which assumed a stiffness decrease for shear walls with no degradation of ultimate strength for lightly and moderately damaged walls. It should be noted in the event where there is heavy damage (failure of structural member), the ultimate strength would need to be reduced which is not considered in this study and needs further investigation. The stiffness of stories 5-6 (case 1), story 3 (case 2), and story 2 (case 3) are assumed to decrease to 80% and 60% of their original stiffness to simulate the condition when damage occurred on the specified floor(s) as shown in Fig 12(b). For each case, sensors location as groups 2.B, 3.A and 3.B are used, and the results after damage are compared with the results of the undamaged model. It should be noted, that optimum location of sensors and expected error could differ in case of a concentration of severe damage (stiffness degradation), which is evaluated in following sections of this study. The same mode shape is used for the mode method estimation, even for damaged structures. Future studies are needed to update the effect of damage on mode shape.



(a) (b)

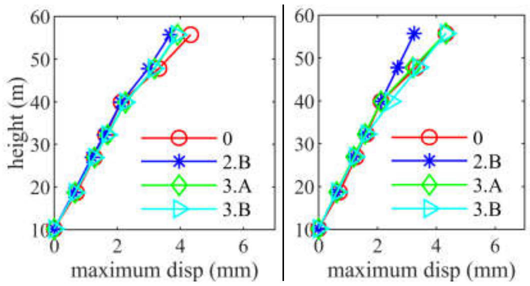
Fig.12 Story cases (a) assumptions for stiffness degradation (b) degradation cases

4.1. Case 1

In this case, the stiffness of stories 5-6 are assumed to decrease to 80% and 60% of the original stiffness to simulate a damaged condition.

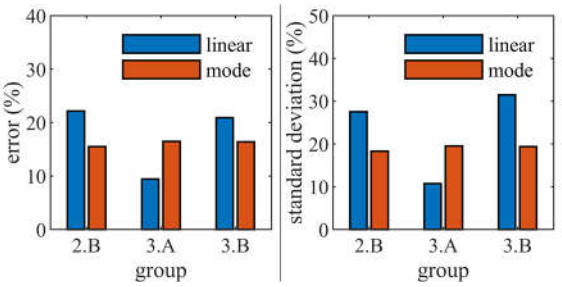
(1) Decrease to 80%

The results of the stiffness of stories 5-6 reduced to 80% are shown in Fig. 13 and 14. For linear estimation, the error of the model with stiffness degradation was approximately twice as large as the error of the model with no stiffness degradation for each group. For mode estimation, the error is larger 7 times for each group compared to the result of the undamaged structure. Since the mode shape due to the stiffness degradation is different, the error for the mode estimation increased.



(a) (b)

Fig.13 Estimated Maximum Displacement (a) mode estimation (b) linear estimation

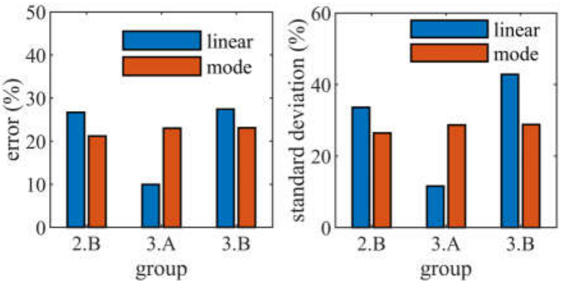


(a) (b)

Fig.14 Case 1, 80% (a) error (b) SD

(2) Decrease to 60%

The results of the stiffness of stories 5-6 reduced to 60% are shown in Fig 15. Compared with the result before damage, for each group, the error increased approximately 3 times for linear estimation using the damaged model, and approximately 10 times for mode estimation.



(a) (b)

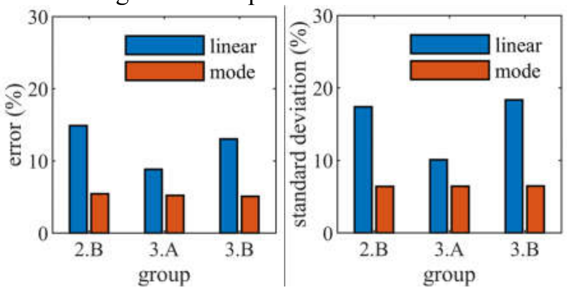
Fig.15 Case 1, 60% (a) error (b) SD

4.2. Case 2

In this case, the stiffness of story 3 is assumed to decrease to 80% and 60% of the original stiffness to simulate damage at midheight of the structure. Because the focus here is on quantifying error, the displacement-height curve is not displayed in the following sections.

(1) Decrease to 80%

The results of the stiffness of story 3 reduced to 80% are shown in Fig. 16. For linear estimation, the error increased approximately 1.5 times for each group compared to the undamaged model. For mode estimation, the error increased approximately 2.4 times for each group. The error associated with case 2 is smaller than the error associated with case 1, especially for mode estimation, which indicates that the concentration of damage at mid-height of a structure may cause less error in estimating the fundamental mode shape than damage accumulating near the top of the structure.

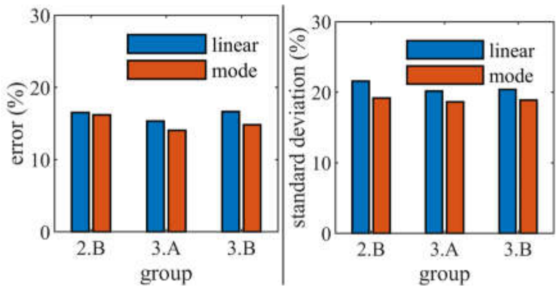


(a) (b)

Fig.16 Case 2, 80% (a) error (b) SD

(2) Decrease to 60%

The results of the stiffness of story 3 reduced to 60% are shown in Fig. 17. For linear estimation, the error increased approximately 1.8 times for each group compared to the undamaged model. For mode estimation, the error increased approximately 8 times for each group. The trend of the SD is similar to the trend in error. For case 2, group 3.A and 3.B resulted in larger error than case 1 because the 3F was severely damaged.



(a) (b)

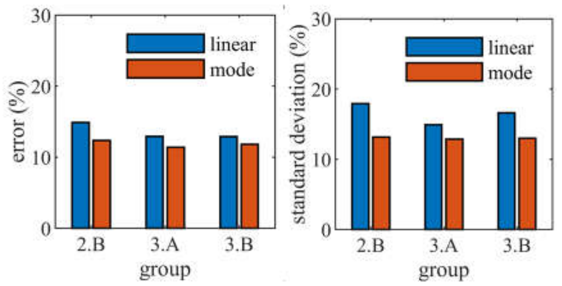
Fig.17 Case 2, 60% (a) error (b) SD

4.3. Case 3

In this case, story 2 is assumed to decrease to 80% and 60% of the original stiffness to simulate damage at the midheight of the structure.

(1) Decrease to 80%

The results of the stiffness of story 2 reduced to 80% are shown in Fig. 18. Compared to the undamaged model, the error increased approximately 1.4 times and 6 times for linear and mode estimation.

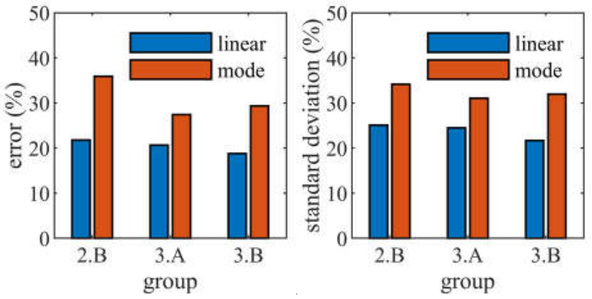


(a) (b)

Fig.18 Case 3, 80% (a) error (b) SD

(2) Decrease to 60%

The results of the stiffness of story 2 reduced to 60% are shown in Fig. 19. Compared to the original model, the error increased approximately 2.2 times and 15 times for linear and mode estimation.



(a) (b)

Fig.19 Case 3, 60% (a) error (b) SD

4.4. Summary of the Results

Comparisons of the increase in error and SD when considering stiffness degradation (damaged model) are shown in Table 2. Damage simulated in case 3 (story 2) results in the smallest increase in error compared to case 1 (stories 5-6) and case 2 (story 3). The mode method results in larger error than the linear method because the mode shape is not accurate when stiffness degradation occurs. The effect of stiffness degradation on mode estimation is about 4 times greater than linear estimation. Updating the original mode based on known locations of damage or sensor data is likely to improve the results. Such an iteration to the mode shape in practice using a limited number of sensors needs further investigation.

Table 2 Increase in error between damaged model and original undamaged model

Condition		Linear		Mode	
Case	Stiffness decrease	error	SD	error	SD
Case 1	80%	2	1.5	7	6.5
	60%	3	2	10	8
Case 2	80%	1.5	1.1	2.5	2.5
	60%	1.8	1.3	8	6.5
Case 3	80%	1.4	1.1	6	4
	60%	2.2	1.3	15	11

5. CONCLUSIONS

This paper utilized two methods, linear and mode, to estimate the story displacement response of a NPS model. The following conclusions are drawn:

a) *Accuracy of mode and linear estimations:* For an undamaged six-story structure evaluated in this study, the tendency of the results shows that the mode method has a good estimation of displacement with only 2 sensors. The accuracy of the mode method is 2% and for the linear method is 15%. The accuracy of the linear method rapidly increased when the number of sensors increased.

b) *Optimum location and number of sensors:* For the case of using two sensors applied to the undamaged six-story model, the results show that one sensor at 1F and one sensor at 5F produced the smallest error. While in the case of having three sensors, one at 1F, 5F, and 7F produced the smallest error.

c) *Assuming damage location (parametric stiffness degradation of floors)*: Error in the estimation of displacement using the mode shape increases when considering stiffness degradation. Because the mode shape used for the mode estimation is not accurate anymore due to the damage, the linear method shows relatively better results than the mode method.

ACKNOWLEDGEMENT

A part of this study was carried out in the Joint Research Project "Research and Development for the Decommissioning of Fukushima Daiichi Nuclear Power Plants" between Tohoku University and TEPCO Holdings, Inc. Fukushima Daiichi D&D Engineering Company.

REFERENCES

[1] Xu, K., and Akira M., "Estimation of maximum drift of multi-degree-of-freedom shear structures with unknown parameters using only one accelerometer," Structural Control and Health Monitoring, May. 2021, pp.1-20 (28:e2799).

[2] Shegay A., Tabata Y., Fujita K. et al., "Estimating the Seismic Response of Buildings in Future Earthquakes Based on Past Accelerometer Data", 17th WCEE, 2020, Sendai, Japan.

[3] Kikuchi, Y., Nashimoto, Y., Suzuki, Y., and Maeda, M., "A Study on the Correspondence Estimation Method for Unobserved Layers in High-Rise RC Buildings Using a Small Number of Acceleration Sensors", Proc. of the JCI, 38 (2), 2016, pp.991-996. (in Japanese)

[4] Pan, H., Yuen, K., and Koichi, K., "Displacement Estimation for Nonlinear Structures Using Seismic Acceleration Response Data," Journal of Earthquake Engineering, Nov. 2021, pp. 1-19.

[5] Fukushima Nuclear Accident Independent Investigation Team Report. https://www.tepco.co.jp/decommission/information/accident_investigation/pdf/120620j0306.pdf. (in Japanese)

[6] Datta, T., "Seismic analysis of structures". John Wiley & Sons, Ltd. 2010, pp. 369

[7] Hamood, A., Maeda, M., Ogata, Y. et al., "Residual seismic performance of damaged reinforced concrete walls," Engineering Structures, Vol. 243, Jun. 2021.