EFFECT OF BEAM AXIAL DEFORMARION ON COLUMN SHEAR IN REINFORCED CONCRETE FRAMES

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SUMMARY

A computer program for nonlinear analyses of reinforced concrete frames was developed, in which beam axial deformations due to nonlinear cyclic loading were incorporated. The beam model was verified in detail through beam tests specially conducted with axial restraint force applied in proportion to the beam axial elongation. Nonlinear pushover analyses of reinforced concrete frame structures with different parameters, such as beam depth, number of spans, and number of stories, were carried out with and without considering beam axial deformations. Effects of beam elongation on column shear were investigated both analytically and theoretically. A simple and practical method for estimating magnification of the column shear due to the beam axial deformation was presented.

INTRODUCTION

It is desirable that reinforced concrete frame buildings form a ductile overall collapse mechanism, in other word, the weak beam-strong column type mechanism during a severe earthquake. In design analysis, the responses of buildings, especially shear forces in the columns, should precisely be estimated to ensure the intended overall beam-yielding mechanism. Therefore, the actual behavior of the structure during an earthquake has been investigated from various viewpoints, such as overstrength of beam, dynamic magnification or two-way action. These effects have been taken into account as simple design formula in recent design code or guidelines in addition to the conventional and definitive design method based on equivalent static loading.

It has been pointed out analytically or experimentally that the beam axial deformations caused by the material properties of reinforced concrete members could change the column responses significantly [Takiguchi et al, 1977][Wada et al, 1990]. Nevertheless, the effects of the beam axial deformations has been neglected in practical design analysis, even by the most sophisticated nonlinear analytical method. Generally, the nodal lateral displacements in a floor are reduced to those of representative point assuming in-plane rigidity of the slab for the efficiency of calculation, so that the beam axial deformation can not be incorporated.

In this study, a computer program was developed for nonlinear analyses of reinforced concrete frames, in which the beam axial deformations due to nonlinear cyclic loading could be simulated. On the other hand, four beams were tested under axial restraint force applied in proportion to the axial elongation, by which the analytical modeling for beam was verified. The effects of the beam elongation on the responses, especially shear forces in the first-story exterior columns, were investigated through analyses of reinforced concrete frames with different beam depth, different number of spans, and different number of stories. A simple and practical design method for estimating magnification of column shear was presented based on the mechanical properties of frames.

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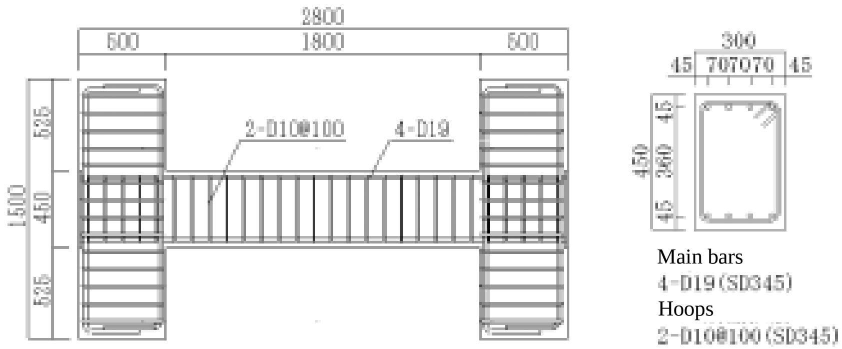
VERIFICATION OF ANALYTICAL MODEL

Member Model for Beam:

The beam axial deformation, that is, the elongation occurs during inelastic cyclic loading, even under compressive axial force, due to the material properties of reinforced concrete including crack opening behavior. In this study, a fiber model based on the material properties of concrete and steel was used for the beam members. The inelastic flexibility of plastic zones at the two ends was evaluated by the fiber model under the applied moments and axial force based on the stress-strain relationships of concrete and steel elements. The flexibility matrices of members were formulated by the integration of assumed flexibility distribution for bending and axial deformation along the member length. Therefore, the shapes of these flexibility distributions were investigated through reinforced concrete beam tests.

Outline of Beam Tests:

Four beam specimens of one-half scale model were tested under anti-symmetric bending and axial restraint force applied in proportion to the measured beam axial elongation [Bunno et al, 1999]. The stiffness constant for the axial force was selected as 100 tonf/cm or 400 tonf/cm, representing the lateral restraint stiffness of columns in prototype frame structures. The shear span ratio was 1.0 or 2.0. The inelastic behavior of the beam model was compared with the observed from one of the specimens with 100tonf/cm restraint and shear span ratio of 2.0. The details of the specimen are illustrated in Figure 1.



σ_B	Stiffnes at $1/3\sigma_B$
382.0	2.88×10^5

σ_B□compressive strength

Bar	E_s	σ_y
D10	1.623×10^6	3693
D19	1.826×10^6	3813

E_s□Young coefficient

□Unit□kgf/cm^2□

Figure 1: Details of the beam specimen

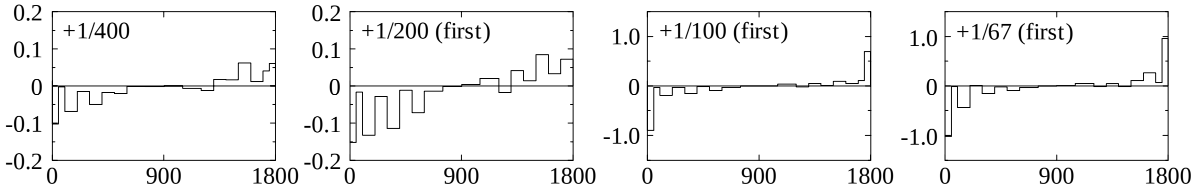
Flexibility Distribution:

The flexibility distributions for bending and axial deformation of reinforced concrete members were investigated through the test result. Figures 2 and 3 shows the distribution of curvature and axial strain along the span length of the specimen at the peak rotation amplitudes of 1/400, 1/200, 1/100, and 1/67 in the positive first cycle of loading. Both curvature and axial strain at the two ends became relatively larger with the inelastic behavior, which was observed significantly after yielding of the main bars (1/100). The beam member consists of plastic zones at the ends and intermediate elastic element. To idealize these observed flexibility distributions after yielding, the length of each plastic zone was assumed to be 10cm for the beam depth of 45cm.

Analysis of the test:

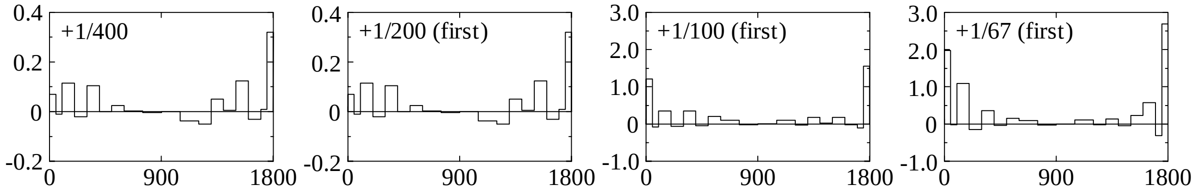
The beam test was simulated using the same member model as used in the response analyses of frames. The axial force in the analysis was applied in proportion to the elongation of the analytical model as was controlled in the test. The fiber slices at the ends of the member consisted of steel elements at its location and nine concrete elements divided along the beam depth. The hysteresis models of stress-strain relationship for concrete and steel are shown in Figure 4 and Figure 5. The material properties from the test were used to determine each parameter of the hysteresis models. However, the elastic stiffness of steel was made 0.14 times the calculated from the original elastic modulus so that the yield deformation of the specimen could be properly simulated considering bond deterioration, pull-out deformation and shear deformation [Kimura et al, 1999].

The observed and analytical relations between the shear force and the rotation angle are shown in Figure 6. The relations between the axial deformations and the rotation angles are shown in Figure 7. The analytical results in solid lines are compared with the test results in dashed lines until the deformation amplitudes up to 1/67, because bond splitting failure occurred after 1/50 of loading. Pinching behavior was observed in the test while a spindle-shaped hysteresis was obtained from the analysis, because the slip behavior in the test might be caused by the loss of bond which was neglected in the analysis. However, the peak strength of the specimen as well as the peak axial elongation in the test were simulated well by the analytical model.



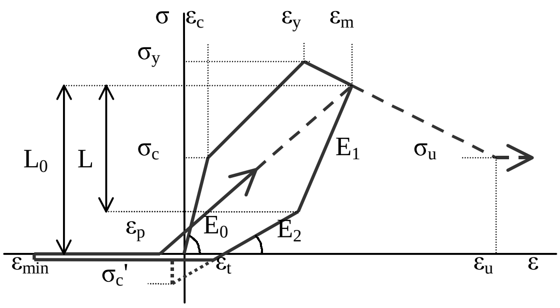
Vertical axis : Curvature (x 10^{-4}/mm) , Horizontal axis : Length (mm)

Figure 2: Distributions of the curvature in the beam test



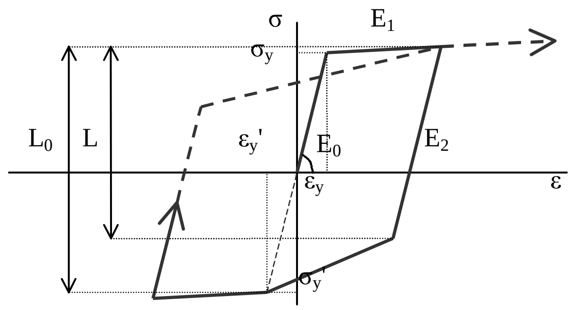
Vertical axis : Axial strain (x 10^{-2}) , Horizontal axis : Length (mm)

Figure 3: Distributions of the axial strains in the beam test



$L = 3/4 \times L_0$ $E_1 = E_0 \times (\varepsilon_y / \varepsilon_m)^{1/2}$

Figure 4: Hysteresis model for concrete element



$L = \beta \times L_0$ $E_1 = 0.001 \times E_0$ $E_2 = E_1$

Figure 5: Hysteresis model for steel element

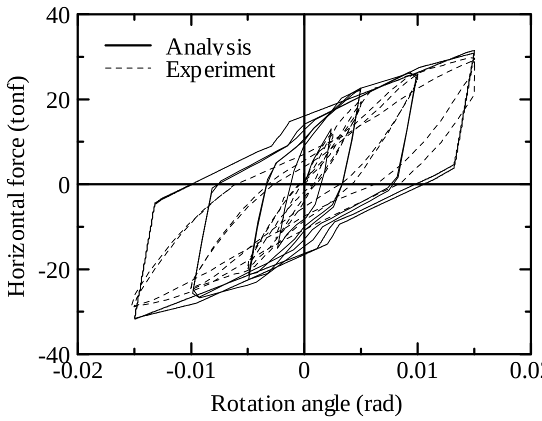


Figure 6: Observed and calculated hysteresis realtions

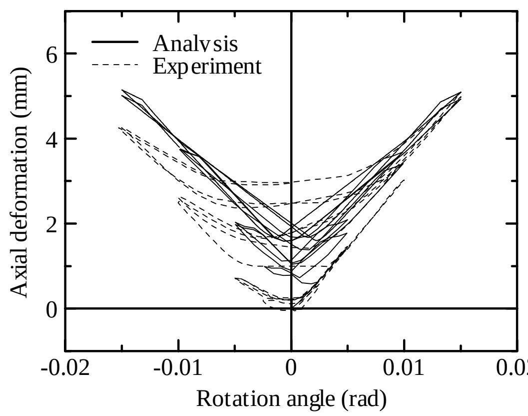


Figure 7: Observed and calculated axial deformations

ANALYSES OF REINFORCED CONCRETE FRAMES

Analyzed Structures:

Nine plane frames with different structural configurations were designed and analyzed as follows. (a) Beam depth: 4-span and 4-storied frames were designed with three types of the beam depths as 40cm, 80cm, and 120cm. (b) Number of spans: three frames with different number of spans as 4, 8, and 12 were analyzed in case of 120cm beam depth and 4-stories. (c) Number of stories: number of stories was varied as 2, 4, and 6 in case of 80cm beam depth and 8-spans. The story height was 3.5m and the span length was 6m. The column section was uniformly 60×60cm and the beam width was 30cm. Gravity axial load of 10tonf and 20tonf were given on the exterior and interior nodes respectively. The bending strengths of the beam sections were assumed to be equal to those from the elastic analysis under seismic loading, while the bar arrangement in the columns was designed with the magnification factor of 1.5, so that the beam-yielding mechanism would be ensured. However, the maximum area of reinforcement in each story was assumed as each representative value. Static pushover analyses of these frames were carried out under an inverted triangular distribution of seismic loads, which were distributed on each node in a floor in proportion to its tributary floor area.

Effects of Structural Configuration on Beam Deformations:

Figure 8 shows the analytical relations between the shear forces of the first-story exterior column on the compressive side and the overall drift for the frames with different beam depths of 40cm, 80cm, and 120cm. The drift was defined at the center column, The relation from the analysis neglecting the beam axial elongation was also shown in the figure for the frame with 120cm beam depth. The column shear force became much higher than that in case of neglecting beam deformation. The increment is larger in case of deep beam model, which increases with the story drift. Figure 9 shows the axial deformation of the beam connected to the first-story exterior column, which increases with the overall rotation angle. The axial deformation is roughly in proportion to the lateral drift as well as the beam depth. The axial deformation was expressed by the ratios to the length of the member except for the rigid zones. The axial deformations of each beam increased in proportion to the beam depth up to the rotation angle of 0.002rad, which was based on the geometric relation of frames. However, the elongation of the beam with the depth of 120cm was much larger than that of the beam with 40cm depth after 0.002rad, because the reinforcement of the former yielded earlier than those of the latter as shown in Figure 9.

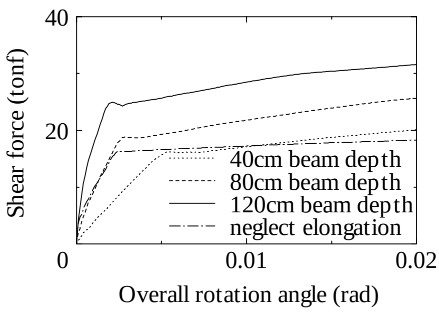


Figure 8: Effect of beam depth on the shear at external column

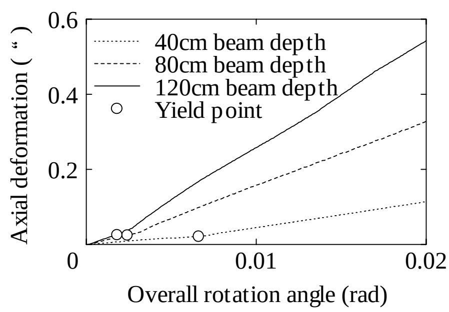


Figure 9: Effect of beam depth on the beam axial deformation in the external span

The number of spans could be another key parameter that controls the increase of shear force of the exterior column. Figure 10 shows the analytical relations between the shear forces of the first-story exterior column on the compressive side and the overall drift for the frames with 4-, 8- and 12-spans. The shear force is higher in case of multi-bay frames, because the cumulative beam axial elongation is larger which induces larger incremental drift to the column. The shear force and the incremental drift of the exterior column are shown in Figure 11. The incremental drift is defined as the difference between the drift of the exterior column and that of the center column in the first-story. The incremental drifts could increase in proportion to the number of spans if the same beam axial elongation is assumed for all beams. However, the increment was not proportional but less. This is because the axial deformations of the interior beams with large number of spans were smaller than those of frames with small number of spans, as shown in Figure 12, which shows the distributions of the axial deformations in the second-floor beams at a rotation angle of 0.01rad from the center span to the exterior span in the compressive side. This was caused by the larger compressive restraining forces against the axial deformations of the interior beams in the multi-bay frame.

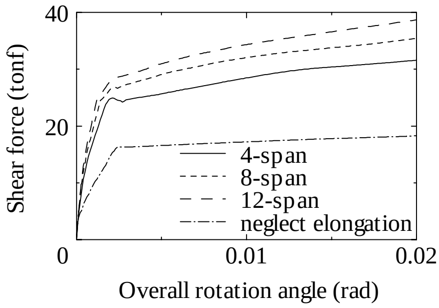


Figure 10: Effect of number of spans on the shear at external column

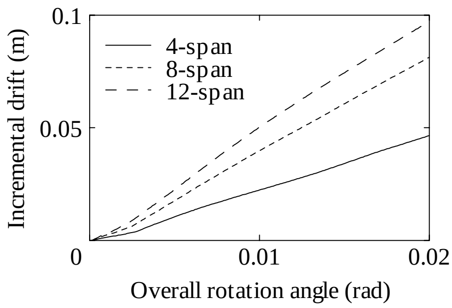


Figure 11: Effect of number of spans on the beam axial deformation in the external sapn

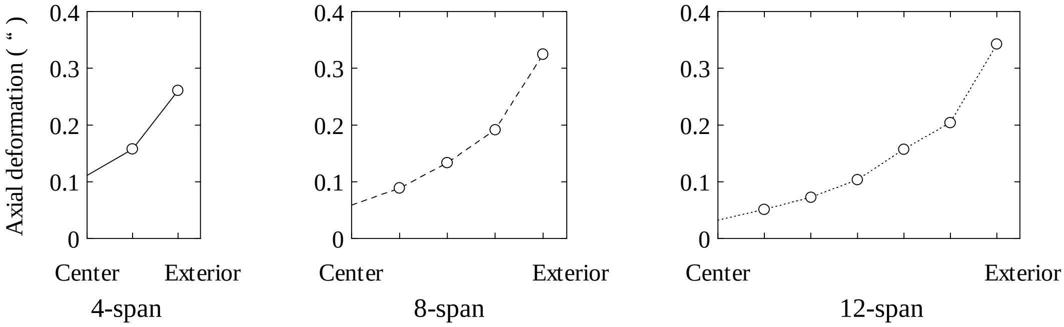


Figure 12: Axial deformations in the second-floor beams

The distributions of the beam axial forces in the second-floor along all bays are shown in Figure 13 for the frames with different number of stories. The beam axial forces were large in the middle spans until yielding of the frames. The axial forces of the beams in the right end became larger than those in the middle after yield in four- and six-story frames. Although the incremental drifts of higher frame were smaller, the beam axial forces were larger. This was caused by the larger restraint against the beam axial elongation due to the first-story and upper-story columns of the higher frame. The column shear force of the higher frame would be larger, because the axial force of the beam connected to the exterior column is nothing but the incremental shear force.

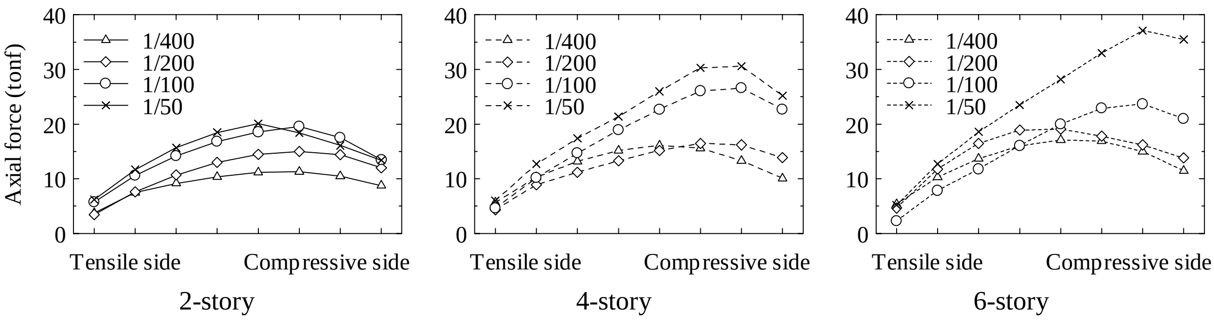


Figure 13: Axial force distributions in the second-floor beams

EVALUATION OF COLUMN SHEAR FORCES

Estimation of Beam Axial Elongation:

It might still be difficult to carry out in practical design above sophisticated analysis incorporating axial deformations in beams. A method of evaluating the incremental column shear forces due to the beam axial deformations was developed for the practical design procedure. The incremental drift of the columns must be estimated to evaluate the incremental shear considering beam axial deformations. The drift increment of a column should be defined as the difference from the drift of the reference column. To make the problem simple, only the first-story columns were considered below, because the incremental shear forces were much larger generally in the first story due to high restraint force from foundation beams.

For one-span frame, if inelastic axial deformation of beam can be derived from simple rigid body model shown in Figure 14(a) by neglecting compressive strain of concrete, the relation between drifts of both side columns can be formulated as equation (1).

$$r_2 = \frac{1}{1 - D/H} \times r_1 \tag{1}$$

As shown in Figure 14(b), this relation could be extended for a multi-span model as equation (2).

$$R_n = \left(\frac{1}{1 - D/H} \right)^n \times R_0 \tag{2}$$

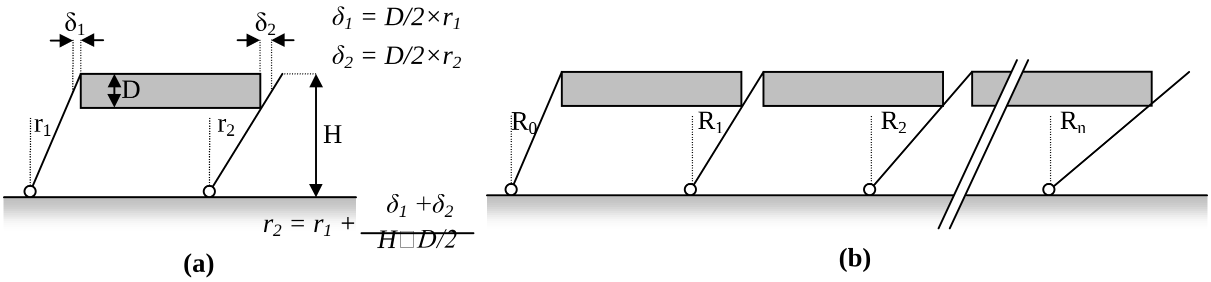


Figure 14: Simplified rigid body model for plastic beam elongation

Above estimation is simple and conservative, which may be used in practical design. However, the incremental drift directly affects the accuracy of the estimation of the incremental shear. In this study, a more detailed model [Bunno et al, 1999] was also used to estimate the beam axial deformations under axial restraint force. The model is based on the flexural theory of the section at the ends, from which the axial elongation including compressive strain along the member length is approximated. The compressive axial deformation δ_c was subtracted from the rigid body deformation by equation (4) to derive the beam axial deformation as equation (3). The axial compressive deformation due to the axial restraint was given by equation (5) based on the equilibrium of axial force considering the stiffness of diagonal strut and the stiffness constant of external force acting on the beam.

$$\delta_{ax}' = \delta_{ax} - \delta_c \tag{3}$$

$$\delta_{ax} = D\theta \tag{4}$$

$$\delta_c = \frac{-2KL_c + 2\sqrt{(KL_c)^2 + KE_cL_cbD}}{E_cb}\theta \tag{5}$$

where, δ_{ax}': axial elongation considering the compressive deformation, δ_{ax}: axial elongation from rigid body, δ_c: compressive deformation of the strut, D: depth of beam, θ: rotation angle of beam bending deformation, L_c: length of the strut, K: stiffness for restraint force, b: width of beam, E_c: Young's modulus of concrete.

The incremental drifts of the first-story exterior columns from the equations (3) and (4), and the sophisticated frame analysis were compared in Figure 15 in case of the frame with 80cm beam depth, 8-span, and 4-story. The drift from equation (3) was estimated through the relation between the shear force and the rotation angle of the first-story center column from the analysis without considering beam axial deformations (Figure 16), based on the assumption that the beam axial elongation was restrained only by the first-story columns. The estimation from equation (3) was more precise than that from equation (4) because the compressive deformation of strut was considered. The drifts by both estimations were a little larger than those by the frame analysis. This was due to the assumption that the only first-story columns restricted the beam axial elongation.

The method by equation (3) gave a good estimation for the frame analysis as well as the test results. However, it requires stiffness for the axial restraint force, which shall be determined from column stiffness for each beam. It might be too much sophisticated as practical design formula. Simple estimation from equation (2) or (4) might be enough for design practice.

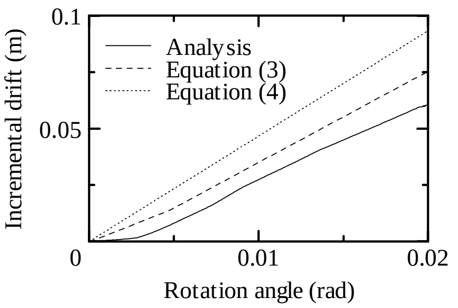


Figure 15: Analytical and estimated incremental drift at external column

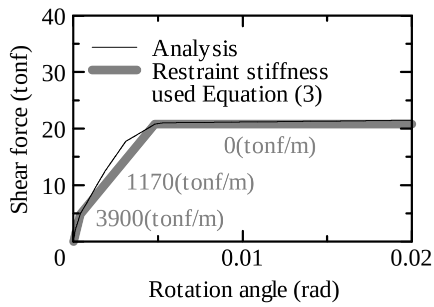


Figure 16: Restraint stiffness of column for detailed estimation

Evaluation of Incremental Column Shear Force:

By assuming the axial elongation of the beams as above, the incremental shear force of the exterior column in the first story was approximated as below. At the first stage, the column shear forces may be assumed to be identical or can be derived from pushover analysis without consideration of the beam axial elongation, as shown in Figure 17(a). However, due to the elongation of the beams and the stiff foundation, the additional lateral incremental drift is imposed on the columns apart from the center of the frame, as shown in Figure 17(b), which induce the incremental shear forces, especially on the exterior column. After yielding of column base, the incremental shear forces could be estimated by assuming that horizontal forces are subjected to the cantilever column as shown in Figure 17(c).

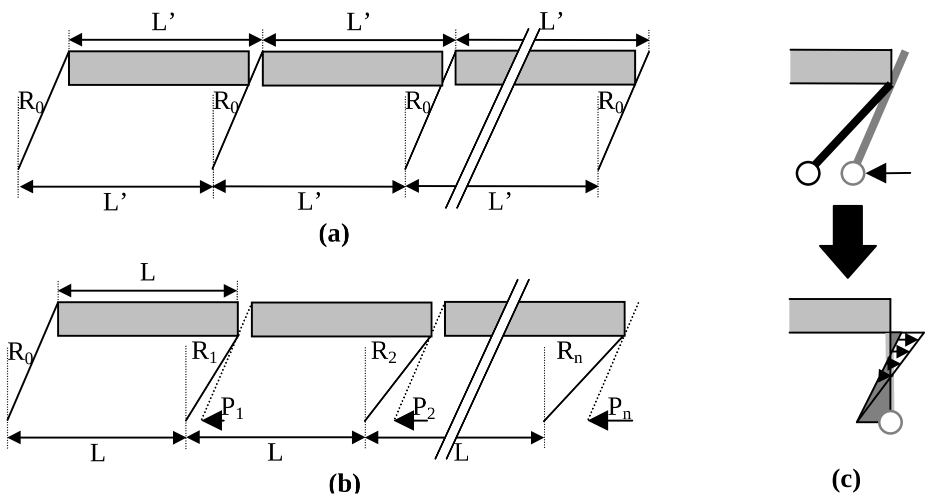


Figure 17: Method of estimating incremental column shear due to beam axial deformation

A numerical example is shown for the analysis of 80cm beam depth, 8-span and 4-story frame. The rotation angle of the exterior column was 0.0052(rad.) when the exterior column yielded, in case of the analysis without considering the beam axial elongation. The bending moment of the column top was 12.0 (tonf.m) at the same stage. The incremental drift of 0.016(m) was subjected to the exterior column in case of the analysis with considering the beam axial elongation from above estimation. The incremental moment for this incremental drift was 36.8(tonf.m) from Figure 18, which shows the relation between the bending moment and the rotation angle of the cantilever column. In terms of shear force, the increment is estimated to be 11.9(tonf).

Figure 19 shows the relations between the shear force of the first-story exterior column and the rotation angle of the first-story center column from the analyses with and without considering the beam axial elongation. The one component model, with Trilinear hysteresis model was used for beams in the analysis without considering the axial elongation. The responses in case of the analysis considering the beam axial elongation evaluated as above was also plotted in Figure 19. Although the estimated shear force was a little large compared with the shear force at the same drift from the sophisticated analysis, the response was simulated well by the simplified estimation.

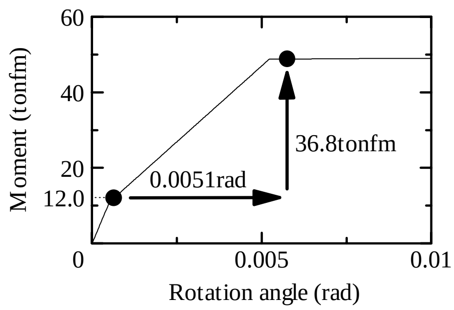


Figure 18: Incremental shear of cantilever column due to incremental drift

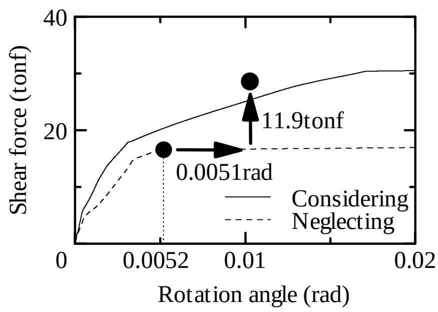


Figure 19: Estimation of incremental shear of external column due to beam axial deformation

CONCLUSIONS

A computer program for nonlinear analyses of reinforced concrete frames was developed, in which beam axial deformations could be simulated by using the fiber model based on the material properties of concrete and steel. The beam model used in this study was verified through the analyses of the beam tests, which were specially conducted with axial restraint force applied in proportion to the beam axial elongation.

The effects of the structural configuration on the responses were investigated through analyses of the plain frames with different beam depth, different number of spans, and different number of stories. The beam axial elongation increased more than the ratios of the beam depth. The incremental drifts of the first-story exterior columns in case of multi-spans were larger but not proportional to the number of spans. The axial forces of the second-floor beams were large in case of the higher frame because of higher restraint forces by beams and columns in upper stories and stiff foundation.

The relations between the inter-story drift and the beam axial elongation could be approximated by a simple model, from which a method of evaluating the incremental shear in the first-story exterior column due to the elongation was presented.

ACKNOWLEDEGMENT

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